

SAMPLE — GEOMETRY WEEKLY MATH STUDY PLAN

Similar Triangles and Proportional Reasoning

A demonstration of the type of personalized study package prepared from a student's weekly assignments.

Homework Analysis

Suppose this week's school homework contains about 30–35 questions involving similar triangles. Although the assignment may contain many diagrams and numerical variations, most of the questions rely on a small number of recurring geometric ideas.

For this sample, the homework has been organized into six skills:

1. Recognize similar triangles using AA similarity.
2. Match corresponding vertices and corresponding sides correctly.
3. Use proportions to find missing side lengths.
4. Use the scale factor to compare lengths and perimeters.
5. Compare areas of similar figures.
6. Use similar triangles in an indirect-measurement problem.

How to use this plan. Study each worked example carefully. Pay particular attention to the order of the vertices and to which sides correspond. Then try the Targeted Practice problem without looking back. When all six skills feel comfortable, complete the Mastery Check.

Skill 1: Proving Triangles Similar by AA

Two triangles are similar if two angles of one triangle are congruent to two angles of the other. This is the **AA Similarity Criterion**.

Remember that once two pairs of angles match, the third pair must also match because the angles in each triangle sum to 180° .

Worked example. Suppose

$$\angle A = 48^\circ, \quad \angle B = 72^\circ$$

in $\triangle ABC$, and

$$\angle D = 48^\circ, \quad \angle E = 72^\circ$$

in $\triangle DEF$.

Since

$$\angle A \cong \angle D \quad \text{and} \quad \angle B \cong \angle E,$$

we have

$$\triangle ABC \sim \triangle DEF$$

by AA similarity.

The order matters:

$$A \leftrightarrow D, \quad B \leftrightarrow E, \quad C \leftrightarrow F.$$

Targeted practice. In $\triangle PQR$,

$$\angle P = 35^\circ, \quad \angle Q = 65^\circ.$$

In $\triangle XYZ$,

$$\angle X = 80^\circ, \quad \angle Y = 35^\circ.$$

Show that the triangles are similar and write the similarity statement with corresponding vertices in the correct order.

Skill 2: Matching Corresponding Sides

A correct similarity statement tells you exactly which vertices and sides correspond.

If

$$\triangle ABC \sim \triangle DEF,$$

then

$$A \leftrightarrow D, \quad B \leftrightarrow E, \quad C \leftrightarrow F.$$

Therefore,

$$AB \leftrightarrow DE, \quad BC \leftrightarrow EF, \quad AC \leftrightarrow DF.$$

Worked example. Suppose

$$\triangle JKL \sim \triangle MNP.$$

Then

$$J \leftrightarrow M, \quad K \leftrightarrow N, \quad L \leftrightarrow P.$$

Thus

$$JK \leftrightarrow MN, \quad KL \leftrightarrow NP, \quad JL \leftrightarrow MP.$$

A common mistake is to match sides because they appear in similar positions in a drawing. Use the *vertex correspondence*, not the appearance of the picture.

Targeted practice. If

$$\triangle RST \sim \triangle UVW,$$

list the three pairs of corresponding sides.

Skill 3: Finding Missing Side Lengths with Proportions

Corresponding sides of similar triangles are proportional.

If

$$\triangle ABC \sim \triangle DEF,$$

then

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

Worked example. Suppose

$$\triangle ABC \sim \triangle DEF,$$

with

$$AB = 6, \quad BC = 9, \quad DE = 10.$$

Find EF .

Because $AB \leftrightarrow DE$ and $BC \leftrightarrow EF$,

$$\frac{6}{10} = \frac{9}{EF}.$$

Cross-multiply:

$$6(EF) = 90.$$

Therefore

$$EF = 15.$$

Targeted practice. Suppose

$$\triangle ABC \sim \triangle DEF,$$

where

$$AC = 8, \quad BC = 12, \quad DF = 14.$$

Find EF .

Skill 4: Scale Factor and Perimeter

The **scale factor** is the ratio of corresponding lengths.

If every length in one figure is multiplied by k , then the perimeter is also multiplied by k .

Worked example. A triangle has side lengths

$$5, \quad 7, \quad 9.$$

A similar triangle has scale factor 3 relative to the first triangle.

Its side lengths are

$$15, \quad 21, \quad 27.$$

The original perimeter is

$$5 + 7 + 9 = 21,$$

and the new perimeter is

$$15 + 21 + 27 = 63 = 3(21).$$

Targeted practice. Two similar polygons have scale factor

$$\frac{5}{2}$$

from the smaller polygon to the larger polygon. If the smaller polygon has perimeter 28, find the perimeter of the larger polygon.

Skill 5: Areas of Similar Figures

Lengths scale by k , but areas scale by

$$k^2.$$

For example, if the side lengths of a triangle are doubled, its area is multiplied by

$$2^2 = 4.$$

Worked example. Two similar triangles have corresponding side lengths 6 and 15. The scale factor from the smaller triangle to the larger triangle is

$$k = \frac{15}{6} = \frac{5}{2}.$$

Therefore the ratio of their areas is

$$k^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

If the smaller triangle has area 32, the larger triangle has area

$$32 \cdot \frac{25}{4} = 200.$$

Targeted practice. Two similar triangles have scale factor 3 from the smaller triangle to the larger triangle. If the smaller triangle has area 14, find the area of the larger triangle.

Skill 6: Indirect Measurement with Similar Triangles

Similar triangles can be used to measure heights or distances that are difficult to measure directly.

Worked example. A 6-foot person casts an 8-foot shadow. At the same time, a tree casts a 28-foot shadow. Assuming the sun's rays make the same angle with the ground, the person and the tree form similar right triangles.

Let h be the height of the tree. Then

$$\frac{6}{8} = \frac{h}{28}.$$

Cross-multiply:

$$8h = 168.$$

Thus

$$h = 21.$$

The tree is 21 feet tall.

Targeted practice. A 5-foot vertical post casts a 7-foot shadow. At the same time, a building casts a 42-foot shadow. Find the height of the building.

Mastery Check

Try these problems without looking back at the worked examples.

1. In $\triangle ABC$,

$$\angle A = 42^\circ, \quad \angle B = 73^\circ.$$

In $\triangle DEF$,

$$\angle D = 65^\circ, \quad \angle E = 42^\circ.$$

Show that the triangles are similar and write a correct similarity statement.

2. If

$$\triangle KLM \sim \triangle PQR,$$

write the three pairs of corresponding sides.

3. Suppose

$$\triangle ABC \sim \triangle DEF,$$

with

$$AB = 9, \quad AC = 12, \quad DE = 15.$$

Find DF .

4. Two similar triangles have scale factor

$$\frac{4}{3}$$

from the smaller triangle to the larger triangle. If the smaller triangle has perimeter 36, find the perimeter of the larger triangle.

5. Two similar figures have corresponding side lengths 4 and 10. If the smaller figure has area 24, find the area of the larger figure.
6. A 4-foot signpost casts a 6-foot shadow. At the same time, a flagpole casts a 27-foot shadow. Find the height of the flagpole.

Before You Return to Your School Homework

You are ready to return to the assigned homework if you can:

- recognize AA similarity;
- write similarity statements in the correct vertex order;
- identify corresponding sides from the similarity statement;
- set up a proportion using corresponding lengths;
- distinguish between the scale factor for lengths and the squared scale factor for areas; and
- recognize when an indirect-measurement problem creates similar triangles.

If one of these still feels uncertain, review that skill before continuing with the assigned homework.

PARENT ANSWER KEY

Geometry — Similar Triangles and Proportional Reasoning

Please keep this section separate from the student study materials.

This section is intended for the parent or guardian. It contains the answers to the Targeted Practice problems and the Mastery Check so you can check the student's understanding and identify any topics that may need additional review.

Part I: Targeted Practice Answers

Skill 1. In $\triangle PQR$,

$$\angle R = 180^\circ - 35^\circ - 65^\circ = 80^\circ.$$

Thus

$$P \leftrightarrow Y, \quad Q \leftrightarrow Z, \quad R \leftrightarrow X.$$

Therefore

$$\triangle PQR \sim \triangle YZX$$

by AA similarity.

Skill 2. From

$$\triangle RST \sim \triangle UVW,$$

the corresponding sides are

$$RS \leftrightarrow UV, \quad ST \leftrightarrow VW, \quad RT \leftrightarrow UW.$$

Skill 3. Since $AC \leftrightarrow DF$ and $BC \leftrightarrow EF$,

$$\frac{8}{14} = \frac{12}{EF}.$$

Thus

$$8(EF) = 168,$$

so

$$EF = 21.$$

Skill 4. Perimeters scale by the same factor as lengths:

$$28 \left(\frac{5}{2} \right) = 70.$$

The larger perimeter is 70.

Skill 5. Areas scale by the square of the length scale factor:

$$3^2 = 9.$$

Therefore

$$14(9) = 126.$$

The larger triangle has area 126.

Skill 6. Let h be the building height:

$$\frac{5}{7} = \frac{h}{42}.$$

Thus

$$7h = 210,$$

so

$$h = 30.$$

The building is 30 feet tall.

Part II: Mastery Check Answers

1. In $\triangle ABC$,

$$\angle C = 180^\circ - 42^\circ - 73^\circ = 65^\circ.$$

In $\triangle DEF$,

$$\angle F = 180^\circ - 65^\circ - 42^\circ = 73^\circ.$$

Thus

$$A \leftrightarrow E, \quad B \leftrightarrow F, \quad C \leftrightarrow D,$$

so

$$\triangle ABC \sim \triangle EFD.$$

2. From

$$\triangle KLM \sim \triangle PQR,$$

we have

$$KL \leftrightarrow PQ, \quad LM \leftrightarrow QR, \quad KM \leftrightarrow PR.$$

3. Since $AB \leftrightarrow DE$ and $AC \leftrightarrow DF$,

$$\frac{9}{15} = \frac{12}{DF}.$$

Thus

$$9(DF) = 180,$$

so

$$DF = 20.$$

4. Perimeters scale by $\frac{4}{3}$:

$$36 \left(\frac{4}{3} \right) = 48.$$

The larger perimeter is 48.

5. The length scale factor is

$$\frac{10}{4} = \frac{5}{2}.$$

The area scale factor is therefore

$$\left(\frac{5}{2} \right)^2 = \frac{25}{4}.$$

Hence

$$24 \left(\frac{25}{4} \right) = 150.$$

The larger figure has area 150.

6. Let h be the flagpole height:

$$\frac{4}{6} = \frac{h}{27}.$$

Thus

$$6h = 108,$$

so

$$h = 18.$$

The flagpole is 18 feet tall.

Using the Results

If the student misses a problem, use the corresponding skill to decide what to review before returning to the school assignment. In geometry, a wrong answer often comes from an incorrect correspondence rather than from the arithmetic itself. Check the vertex order and corresponding sides first.