

SAMPLE — ALGEBRA I WEEKLY MATH STUDY PLAN

Linear Equations and Inequalities

A demonstration of the type of personalized study package prepared from a student's weekly assignments.

Homework Analysis

Suppose this week's school homework contains about 30–35 questions on linear equations and inequalities. Many of those questions look different, but they rely on a much smaller set of mathematical skills.

For this sample, the homework has been organized into six skills:

1. Solve one-step and multi-step linear equations.
2. Simplify both sides before solving.
3. Solve equations containing fractions.
4. Recognize equations with one solution, no solution, or infinitely many solutions.
5. Solve and graph linear inequalities.
6. Translate a word problem into a linear equation.

How to use this plan. Study each worked example and then try the Targeted Practice problem without looking back. When all six skills feel comfortable, complete the Mastery Check.

Skill 1: Solving Multi-Step Linear Equations

The goal is to isolate the variable. Whatever operation you perform on one side of an equation must also be performed on the other side.

Worked example. Solve

$$5x - 7 = 3x + 9.$$

Subtract $3x$ from both sides:

$$2x - 7 = 9.$$

Add 7:

$$2x = 16.$$

Divide by 2:

$$x = 8.$$

A quick check gives

$$5(8) - 7 = 33, \quad 3(8) + 9 = 33.$$

Targeted practice. Solve

$$7x + 4 = 3x + 28.$$

Skill 2: Simplifying Before Solving

When parentheses or like terms appear, simplify first. This usually makes the equation much easier to solve.

Worked example. Solve

$$3(2x - 5) + 4 = 2(x + 7) + 7.$$

Distribute:

$$6x - 15 + 4 = 2x + 14 + 7.$$

Combine like terms:

$$6x - 11 = 2x + 21.$$

Subtract $2x$:

$$4x - 11 = 21.$$

Add 11:

$$4x = 32.$$

Therefore

$$x = 8.$$

Targeted practice. Solve

$$4(2x + 3) - 5 = 3(x - 1) + 20.$$

Skill 3: Equations with Fractions

Fractions often disappear if every term is multiplied by the least common denominator.

Worked example. Solve

$$\frac{x}{3} + \frac{x}{4} = 7.$$

The least common denominator is 12. Multiply the entire equation by 12:

$$12\left(\frac{x}{3}\right) + 12\left(\frac{x}{4}\right) = 12(7).$$

Thus

$$4x + 3x = 84,$$

so

$$7x = 84$$

and

$$x = 12.$$

Targeted practice. Solve

$$\frac{x}{2} - \frac{x}{5} = 9.$$

Skill 4: One Solution, No Solution, or Infinitely Many Solutions

Not every linear equation ends with $x = \text{a number}$.

- If you obtain a true statement such as $5 = 5$, the equation has infinitely many solutions.
- If you obtain a false statement such as $5 = 9$, the equation has no solution.
- Otherwise, the equation usually has one solution.

Worked example 1. Solve

$$4(x + 2) = 4x + 8.$$

Distributing gives

$$4x + 8 = 4x + 8.$$

Subtracting $4x$ gives

$$8 = 8.$$

This is always true, so the equation has **infinitely many solutions**.

Worked example 2. Solve

$$3(x - 1) = 3x + 5.$$

Distributing gives

$$3x - 3 = 3x + 5.$$

Subtracting $3x$ gives

$$-3 = 5,$$

which is false. Therefore there is **no solution**.

Targeted practice. Classify each equation as having one solution, no solution, or infinitely many solutions.

(a) $5(x - 2) = 5x - 10$

(b) $2(x + 4) = 2x + 11$

Skill 5: Solving Linear Inequalities

Solve an inequality much like an equation, with one crucial rule:

Multiplying or dividing by a negative number reverses the inequality sign.

Worked example. Solve

$$-3x + 5 > 17.$$

Subtract 5:

$$-3x > 12.$$

Divide by -3 . Because the divisor is negative, reverse the inequality:

$$x < -4.$$

On a number line, use an open circle at -4 and shade to the left.

Targeted practice. Solve and describe the graph of

$$-4x - 3 \leq 13.$$

Skill 6: Turning Words into an Equation

In a word problem, first decide what the variable represents. Then translate the relationships into an equation.

Worked example. A gym charges a one-time registration fee of \$25 plus \$18 per month. After several months, the total cost is \$133. How many months has the person been a member?

Let m be the number of months. Then

$$25 + 18m = 133.$$

Subtract 25:

$$18m = 108.$$

Therefore

$$m = 6.$$

The membership lasted 6 months.

Targeted practice. A streaming service charges a one-time setup fee of \$12 plus \$9 per month. A customer has paid \$75 in total. How many months has the customer used the service?

Mastery Check

Try these problems without looking back at the worked examples.

1. Solve:

$$6x - 11 = 2x + 17.$$

2. Solve:

$$5(2x - 3) + 7 = 3(x + 4) + 8.$$

3. Solve:

$$\frac{x}{4} + \frac{x}{6} = 10.$$

4. Determine whether the equation has one solution, no solution, or infinitely many solutions:

$$3(2x + 5) = 6x + 15.$$

5. Solve and describe the number-line graph:

$$-5x + 7 \geq 22.$$

6. A taxi charges a \$4 starting fee plus \$3 per mile. A ride costs \$31. How many miles was the ride?

Before You Return to Your School Homework

You are ready to return to the assigned homework if you can:

- keep an equation balanced while isolating the variable;
- distribute correctly and combine like terms;
- clear fractions using a common denominator;
- recognize when an equation has no solution or infinitely many solutions;
- remember to reverse an inequality when multiplying or dividing by a negative number; and
- define a variable and translate a word problem into an equation.

If one of these still feels uncertain, review that skill before continuing with the assigned homework.

PARENT ANSWER KEY

Algebra I — Linear Equations and Inequalities

Please keep this section separate from the student study materials.

This section is intended for the parent or guardian. It contains the answers to the Targeted Practice problems and the Mastery Check so you can check the student's understanding and identify any topics that may need additional review.

Part I: Targeted Practice Answers

Skill 1.

$$7x + 4 = 3x + 28$$

gives

$$4x = 24,$$

so

$$x = 6.$$

Skill 2.

$$4(2x + 3) - 5 = 3(x - 1) + 20$$

becomes

$$8x + 7 = 3x + 17.$$

Thus

$$5x = 10,$$

so

$$x = 2.$$

Skill 3.

$$\frac{x}{2} - \frac{x}{5} = 9.$$

Multiply by 10:

$$5x - 2x = 90.$$

Thus

$$3x = 90,$$

so

$$x = 30.$$

Skill 4.

(a) $5(x - 2) = 5x - 10$ simplifies to $5x - 10 = 5x - 10$, so there are **infinitely many solutions**.

(b) $2(x + 4) = 2x + 11$ simplifies to $2x + 8 = 2x + 11$, which gives $8 = 11$. Therefore there is **no solution**.

Skill 5.

$$-4x - 3 \leq 13$$

gives

$$-4x \leq 16.$$

Dividing by -4 reverses the inequality:

$$x \geq -4.$$

The graph has a closed circle at -4 and is shaded to the right.

Skill 6. Let m be the number of months:

$$12 + 9m = 75.$$

Then

$$9m = 63,$$

so

$$m = 7.$$

The customer has used the service for 7 months.

Part II: Mastery Check Answers

1.

$$6x - 11 = 2x + 17$$

gives

$$4x = 28,$$

so

$$x = 7.$$

2.

$$5(2x - 3) + 7 = 3(x + 4) + 8$$

becomes

$$10x - 8 = 3x + 20.$$

Thus

$$7x = 28,$$

so

$$x = 4.$$

3.

$$\frac{x}{4} + \frac{x}{6} = 10.$$

Multiply by 12:

$$3x + 2x = 120.$$

Thus

$$5x = 120,$$

so

$$x = 24.$$

4.

$$3(2x + 5) = 6x + 15$$

simplifies to

$$6x + 15 = 6x + 15.$$

Therefore the equation has **infinitely many solutions**.

5.

$$-5x + 7 \geq 22$$

gives

$$-5x \geq 15.$$

Dividing by -5 reverses the inequality:

$$x \leq -3.$$

The graph has a closed circle at -3 and is shaded to the left.6. Let m be the number of miles:

$$4 + 3m = 31.$$

Then

$$3m = 27,$$

so

$$m = 9.$$

The ride was 9 miles.

Using the Results

If the student misses a problem, use the corresponding skill to decide what to review before returning to the school assignment. The goal is not simply to count correct answers, but to identify which algebra skills are secure and which need another look.